Name: _____

UW Math Circle

Week 11 – Kakuro Puzzles

Kakuro is a puzzle similar to a crossword, but with numbers instead of letters. Your goal is to fill in the empty squares of a grid with the numbers 1 - 9 following these key rules:

- Each row must sum to the number written to its left
- Each column must sum to the number written at its top
- No numbers can be repeated in a row or column

Often, two rows or columns are next to each other, but separated by a gray square. These count as separate rows or columns. Here is an example of a solved puzzle–check for yourself that it follows each rule!

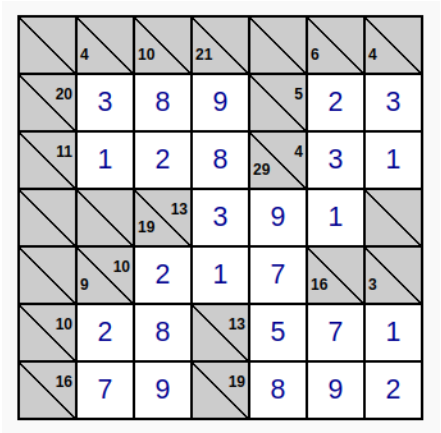
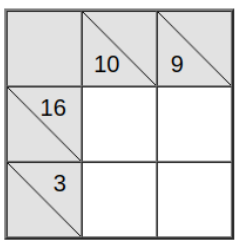
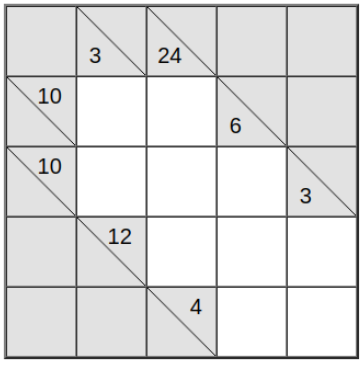


Figure 1: Completed Kakuro Puzzle

1. Try a couple of kakuro puzzles on your own!



Puzzle 1a



Puzzle 1b

1 Starting Strategies

A *kakuro block* is a single row or column in a kakuro puzzle. It is made up of squares and a number which the squares sum up to. To start let's focus on individual kakuro blocks.

2. Is it possible for one of the squares below to be a 1? 2? 3? Why or why not?

3. Is it possible for one of the squares below to be 9? 8? 7? Why or why not?

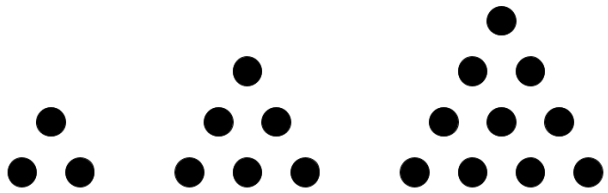
4. Use the above two examples to come up with a general rule for possible solutions to blocks with three squares:

   Bonus: Find a similar rule for blocks of any number of squares.

2 Triangular Numbers

To learn more strategies for solving more challenging kakuro puzzles, let's talk about a useful kind of number.

A *triangular number* is a special number of dots which can be used to form a triangle by stacking like a human pyramid. Take a look at the triangles below as an example:



Three dots can form a triangle that has two dots at the base, six dots can form a triangle that has three dots at the base, and ten dots can form a triangle with four dots at the base. This makes three, six, and ten all triangular numbers.

5. How many dots are in a triangle that has five dots at the base?

6. Is 4 a triangular number? Why or why not?

7. Fill out the table below with the first nine triangular numbers. Is there a pattern in how to find them?

Base	Triangular Number
1	
2	
3	
4	
5	
6	
7	
8	
9	

8. For an individual kakuro block with number and a number of squares, we can try to count all the possible ways to reach the sum. We can always get a new possible solution by rearranging the numbers in our sum, so we will ignore those. Call a valid sum (up to rearranging) a *solution set* to the kakuro block.

   For example, the solution set to the block

   has two solution sets:

$$8 = 5 + 2 + 1$$
$$8 = 4 + 3 + 1$$

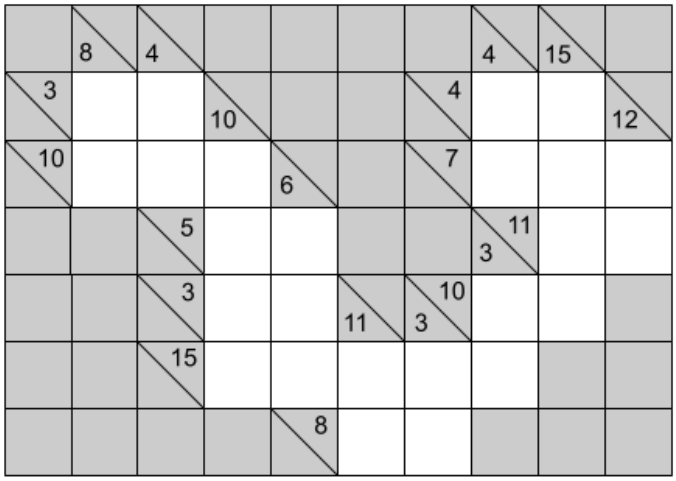
   (a) Check that there are no other solution sets for 8 using three numbers.

   (b) How may solution sets are there for 3 using two numbers? How about 6 with three numbers?

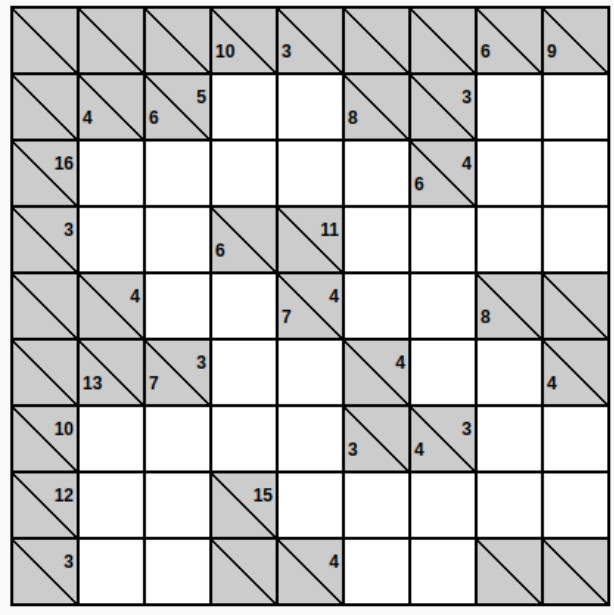
   (c) How many solution sets are there for the nth triangular number using n numbers?

9. Is this kakuro block possible? Why or why not.

10. Use what we've learned about triangular numbers to help you solve some more challenging kakuro puzzles:



Puzzle 11a



Puzzle 11b

3 Kakuro Magic Blocks

Triangular numbers aren't the only numbers with only one way to sum up to them over a certain number of blocks. Any sum/block combination with a unique solution (up to ordering your numbers) we will call a *magic block*.

12. Prove that there is only one way to sum up to 7 over three blocks.

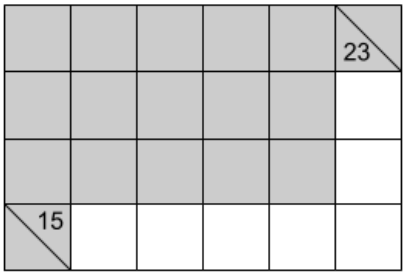
13. There are many more magic blocks that we've listed in the table below. Fill out the table with the unique way to form each sum. Make sure you can explain why each summation is unique.

# of Blocks	Sum	Unique Summation
2	3	$1 + 2$
2	4	
2	16	
2	17	
3	6	$1 + 2 + 3$
3	7	
3	23	
3	24	
4	10	
4	11	
4	29	
4	30	
5	15	
5	16	
5	34	
5	35	

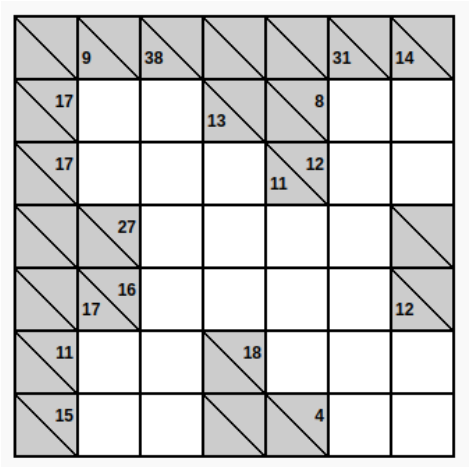
# of Blocks	Sum	Unique Summation
6	21	
6	22	
6	38	
6	39	
7	28	
7	29	
7	41	
7	42	
8	36	
8	37	
8	38	
8	39	
8	40	
8	41	
8	42	
8	43	
8	44	
9	45	

14. Do you notice any patterns in the table above other than the triangular numbers? Describe why these patterns hold.

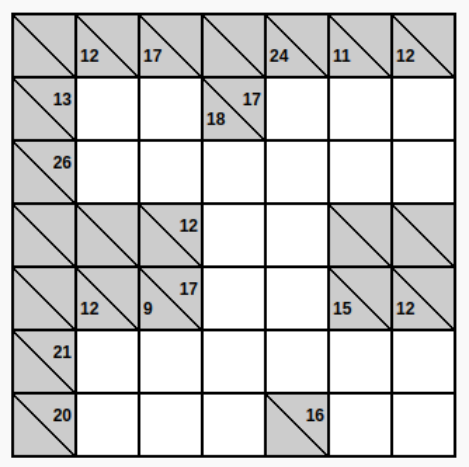
15. Is this kakuro puzzle possible? Why or why not?



16. Use magic blocks to help you solve some more challenging kakuro puzzles:

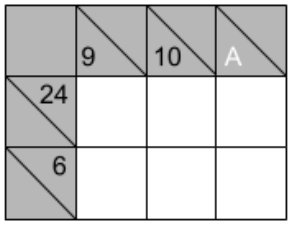


Puzzle 16a

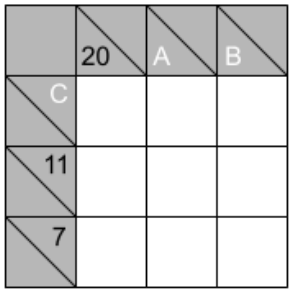


Puzzle 16b

17. For what value(s) of A does the following puzzle have a solution? Bonus: How many different solutions does each value of A have?

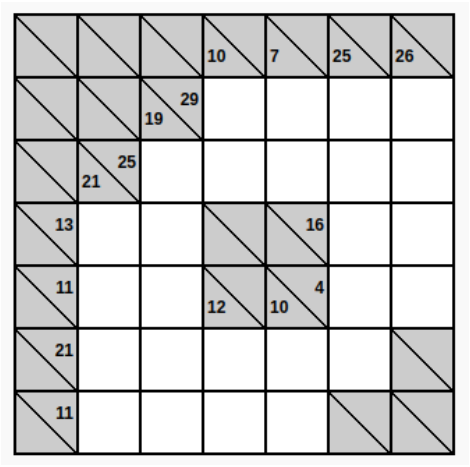


18. Choose values for A and B so that the kakuro puzzle has only one possible solution (including the forced value of C). There are a few choices of A and B that do this. If possible, find them all.

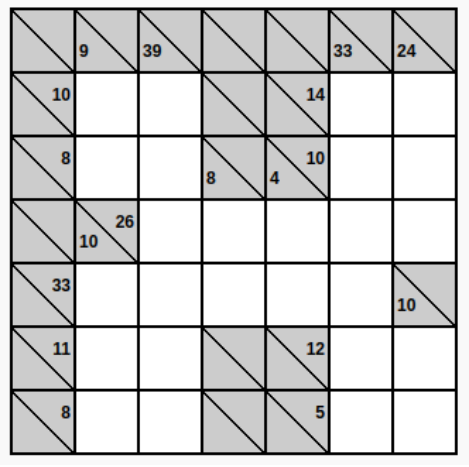


4 More Puzzles

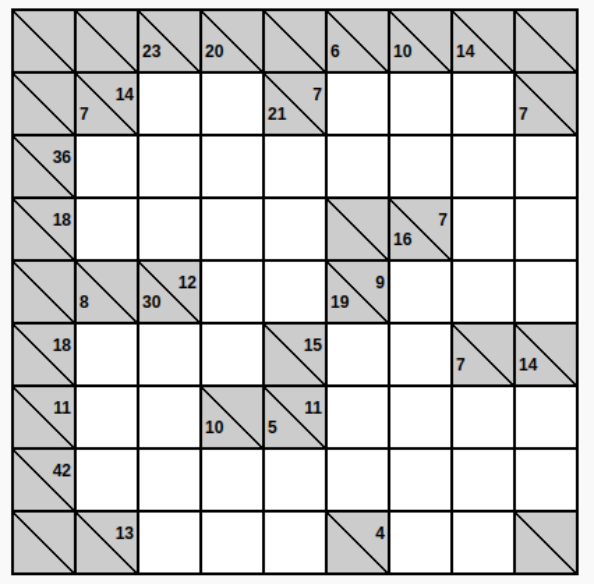
Here are some difficult kakuro puzzles you can work on. As you solve them, search for additional strategies and tricks you can use. Record them here and explain why they work to your group members.



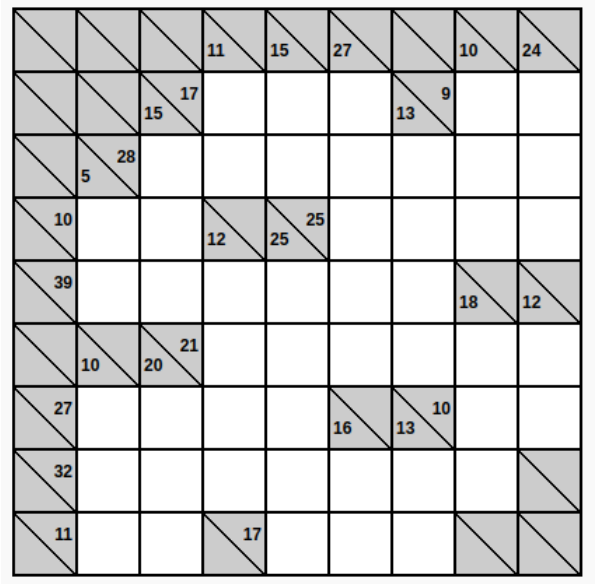
Puzzle 18a



Puzzle 18b



Puzzle 18c

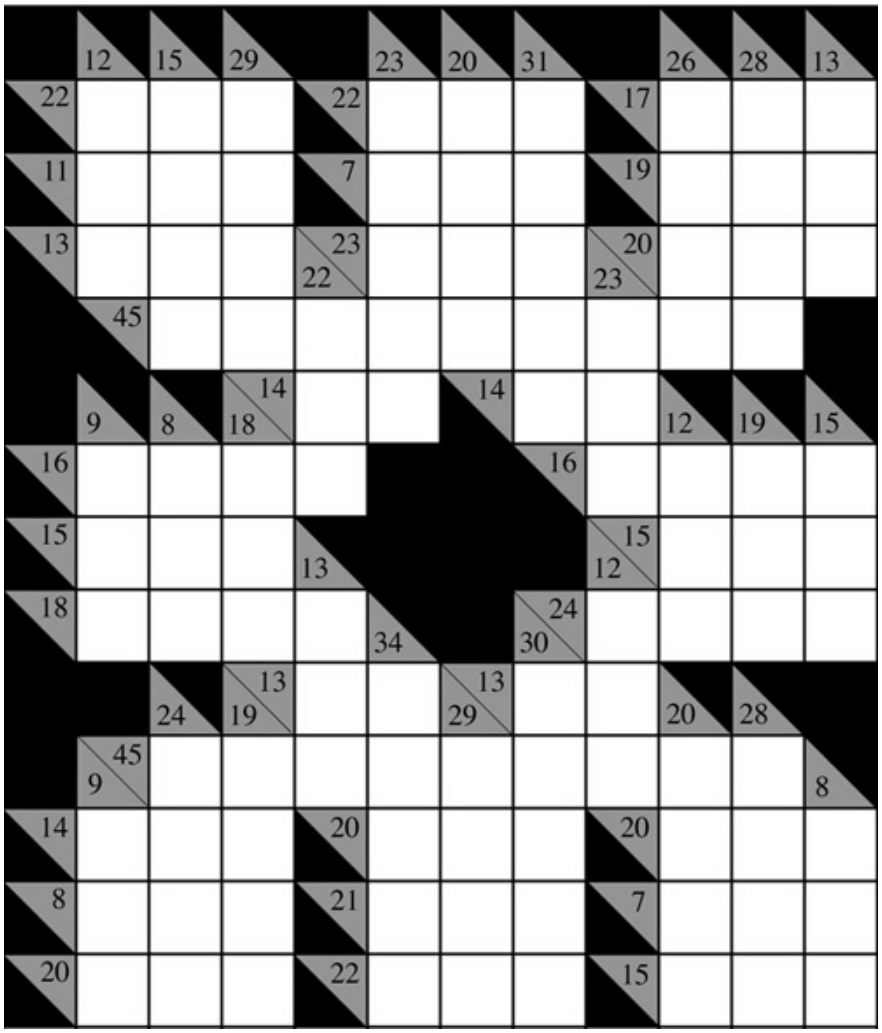


Puzzle 18d

18. What additional tricks did you find? Why do they work?

5 Kakuro Variant

19. For an added challenge, solve this kakuro puzzle using all the traditional kakuro rules, but additionally require that no two adjacent boxes can hold consecutive numbers.



Puzzle taken from post on https://motris.livejournal.com/14366.html